

ECUACIONES DIFERENCIALES GRUPO 13
PRIMERA CLASE EN LÍNEA
OCTUBRE 16, 2025

> restart

>

951)

> Ecua := diff(x(t), t) + 3·x(t) = exp(-2·t)

$$Ecua := \frac{d}{dt} x(t) + 3 x(t) = e^{-2t} \quad (1)$$

> CondIni := x(0) = 0

$$CondIni := x(0) = 0 \quad (2)$$

> with(inttrans)

[addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier, invhilbert, invlaplace, invmellin, laplace, mellin, savetable, setup] (3)

> EcuaTL := subs(CondIni, laplace(Ecua, t, s))

$$EcuaTL := s \mathcal{L}(x(t), t, s) + 3 \mathcal{L}(x(t), t, s) = \frac{1}{s+2} \quad (4)$$

> SolTL := isolate(EcuaTL, laplace(x(t), t, s))

$$SolTL := \mathcal{L}(x(t), t, s) = \frac{1}{(s+2)(s+3)} \quad (5)$$

> SolPart := invlaplace(SolTL, s, t)

$$SolPart := x(t) = e^{-2t} - e^{-3t} \quad (6)$$

> ComprobarUno := simplify(eval(subs(x(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))

$$ComprobarUno := 0 = 0 \quad (7)$$

> CondIni

$$x(0) = 0 \quad (8)$$

> ComprobarDos := subs(t=0, SolPart)

$$ComprobarDos := x(0) = 0 \quad (9)$$

> restart

952)

> Ecua := diff(x(t), t) - 3·x(t) = 3·t³ + 3·t² + 2·t + 1

$$Ecua := \frac{d}{dt} x(t) - 3 x(t) = 3 t^3 + 3 t^2 + 2 t + 1 \quad (10)$$

> CondIni := x(0) = -1

$$CondIni := x(0) = -1 \quad (11)$$

> with(inttrans)

[addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier, invhilbert, invlaplace, invmellin, laplace, mellin, savetable, setup] (12)

> EcuaTL := subs(CondIni, laplace(Ecua, t, s))

(13)

$$EcuaTL := s \mathcal{L}(x(t), t, s) + 1 - 3 \mathcal{L}(x(t), t, s) = \frac{s^3 + 2s^2 + 6s + 18}{s^4} \quad (13)$$

> SolTL := simplify(isolate(EcuaTL, laplace(x(t), t, s)))

$$SolTL := \mathcal{L}(x(t), t, s) = \frac{-s^3 - 2s^2 - 4s - 6}{s^4} \quad (14)$$

Fracciones Parciales

> EcuaUno := rhs(SolTL) = $\frac{A}{s^4} + \frac{B}{s^3} + \frac{D}{s^2} + \frac{E}{s}$

$$EcuaUno := \frac{-s^3 - 2s^2 - 4s - 6}{s^4} = \frac{A}{s^4} + \frac{B}{s^3} + \frac{D}{s^2} + \frac{E}{s} \quad (15)$$

> EcuaDos := lhs(EcuaUno) · s⁴ = expand(rhs(EcuaUno) · s⁴)

$$EcuaDos := -s^3 - 2s^2 - 4s - 6 = s^3 E + s^2 D + s B + A \quad (16)$$

> ParaUno := E = -1

$$ParaUno := E = -1 \quad (17)$$

> ParaDos := D = -2

$$ParaDos := D = -2 \quad (18)$$

> ParaTres := B = -4

$$ParaTres := B = -4 \quad (19)$$

> ParaCuatro := A = -6

$$ParaCuatro := A = -6 \quad (20)$$

> SolTLdos := lhs(SolTL) = subs(A = rhs(ParaCuatro), B = rhs(ParaTres), D = rhs(ParaDos), E = rhs(ParaUno), rhs(EcuaUno))

$$SolTLdos := \mathcal{L}(x(t), t, s) = -\frac{6}{s^4} - \frac{4}{s^3} - \frac{2}{s^2} - \frac{1}{s} \quad (21)$$

> EcuaPartDos := invlaplace(SolTLdos, s, t)

$$EcuaPartDos := x(t) = -(t+1)(t^2 + t + 1) \quad (22)$$

> EcuaPart := invlaplace(SolTL, s, t)

$$EcuaPart := x(t) = -(t+1)(t^2 + t + 1) \quad (23)$$

> restart

955)

> Ecua := 2 · diff(x(t), t) + 6 · x(t) = t · exp(-3 t)

$$Ecua := 2 \frac{d}{dt} x(t) + 6 x(t) = t e^{-3t} \quad (24)$$

> CondIni := x(0) = - $\frac{1}{2}$

$$CondIni := x(0) = -\frac{1}{2} \quad (25)$$

> EcuaBis := $\frac{lhs(Ecua)}{2} = \frac{rhs(Ecua)}{2}$

$$EcuaBis := \frac{d}{dt} x(t) + 3 x(t) = \frac{t e^{-3t}}{2} \quad (26)$$

> with(inttrans) :

> EcuaTL := subs(CondIni, laplace(EcuaBis, t, s))

$$EcuaTL := s \mathcal{L}(x(t), t, s) + \frac{1}{2} + 3 \mathcal{L}(x(t), t, s) = \frac{1}{2(s+3)^2} \quad (27)$$

> SolTL := simplify(isolate(EcuaTL, laplace(x(t), t, s)))

$$SolTL := \mathcal{L}(x(t), t, s) = \frac{-s^2 - 6s - 8}{2(s+3)^3} \quad (28)$$

> SolPart := simplify(invlaplace(SolTL, s, t))

$$SolPart := x(t) = \frac{e^{-3t}(t^2 - 2)}{4} \quad (29)$$

> Ecua

$$2 \frac{d}{dt} x(t) + 6 x(t) = t e^{-3t} \quad (30)$$

> ComprobacionUno := simplify(eval(subs(x(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))

$$ComprobacionUno := 0 = 0 \quad (31)$$

> CondIni

$$x(0) = -\frac{1}{2} \quad (32)$$

> ComprobacionDos := simplify(subs(t=0, SolPart))

$$ComprobacionDos := x(0) = -\frac{1}{2} \quad (33)$$

> restart

961)

> Ecua := diff(x(t), t\$2) + 3*diff(x(t), t) + 2*x(t) = 2*t^2 + 1

$$Ecua := \frac{d^2}{dt^2} x(t) + 3 \frac{d}{dt} x(t) + 2 x(t) = 2 t^2 + 1 \quad (34)$$

> CondIni := x(0) = 4, D(x)(0) = 3

$$CondIni := x(0) = 4, D(x)(0) = 3 \quad (35)$$

> with(inttrans) :

> EcuaTL := subs(CondIni, laplace(Ecua, t, s))

$$EcuaTL := s^2 \mathcal{L}(x(t), t, s) - 15 - 4s + 3s \mathcal{L}(x(t), t, s) + 2 \mathcal{L}(x(t), t, s) = \frac{4}{s^3} + \frac{1}{s} \quad (36)$$

> SolTL := simplify(isolate(EcuaTL, laplace(x(t), t, s)))

$$SolTL := \mathcal{L}(x(t), t, s) = \frac{\frac{4}{s^3} + \frac{1}{s} + 4s + 15}{s^2 + 3s + 2} \quad (37)$$

> SolPart := invlaplace(SolTL, s, t)

$$(38)$$

$$SolPart := x(t) = -3 t - 6 e^{-2 t} + t^2 + 4 + 6 e^{-t} \quad (38)$$

> Ecua

$$\frac{d^2}{dt^2} x(t) + 3 \frac{d}{dt} x(t) + 2 x(t) = 2 t^2 + 1 \quad (39)$$

> CompUno := simplify(eval(subs(x(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))

$$CompUno := 0 = 0 \quad (40)$$

> CondIni[1]

$$x(0) = 4 \quad (41)$$

> CompDos := simplify(subs(t = 0, SolPart))

$$CompDos := x(0) = 4 \quad (42)$$

> CondIni[2]

$$D(x)(0) = 3 \quad (43)$$

> CompTres := D(x)(0) = simplify(subs(t = 0, rhs(diff(SolPart, t))))

$$CompTres := D(x)(0) = 3 \quad (44)$$

> restart

975)

> Ecua := y'' - 4·y = 4·cos(2 x) - $\frac{1}{2}$ ·sin(2 x)

$$Ecua := \frac{d^2}{dx^2} y(x) - 4 y(x) = 4 \cos(2 x) - \frac{\sin(2 x)}{2} \quad (45)$$

> CondIni := y(0) = 0, D(y)(0) = $\frac{1}{8}$

$$CondIni := y(0) = 0, D(y)(0) = \frac{1}{8} \quad (46)$$

> with(inttrans) :

> EcuaTL := subs(CondIni, laplace(Ecua, x, s))

$$EcuaTL := s^2 \mathcal{L}(y(x), x, s) - \frac{1}{8} - 4 \mathcal{L}(y(x), x, s) = \frac{4 s - 1}{s^2 + 4} \quad (47)$$

> SolTL := isolate(EcuaTL, laplace(y(x), x, s))

$$SolTL := \mathcal{L}(y(x), x, s) = \frac{\frac{4 s - 1}{s^2 + 4} + \frac{1}{8}}{s^2 - 4} \quad (48)$$

> SolPart := expand(invlaplace(SolTL, s, x))

$$SolPart := y(x) = -\cos(x)^2 + \frac{\sin(x) \cos(x)}{8} + \cosh(x)^2 \quad (49)$$

> Ecua

$$\frac{d^2}{dx^2} y(x) - 4 y(x) = 4 \cos(2 x) - \frac{\sin(2 x)}{2} \quad (50)$$

> compUno := simplify(eval(subs(y(x) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))

$$compUno := 0 = 0 \quad (51)$$

```
> CondIni[1]
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$$y(0) = 0 \quad (52)$$

```
> CompDos := simplify(subs(x=0, SolPart))
```

$$CompDos := y(0) = 0 \quad (53)$$

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> CondIni[2]
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$$D(y)(0) = \frac{1}{8} \quad (54)$$

```
> CompTres := D(y)(0) = simplify(subs(x=0, rhs(diff(SolPart, x))))
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$$CompTres := D(y)(0) = \frac{1}{8} \quad (55)$$

```
> restart
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Funciones Seccionalmente Continuas

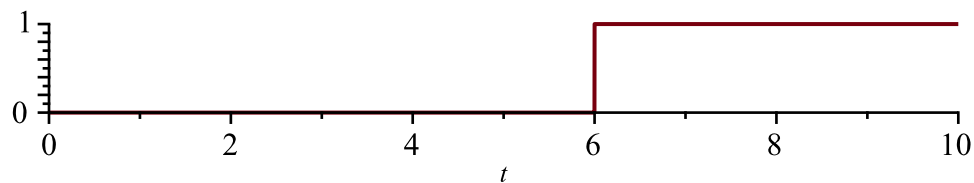
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Función Escalón Unitario

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> EscalonUnitario := Heaviside(t - 6)
```

$$EscalonUnitario := Heaviside(t - 6) \quad (56)$$

```
> plot(EscalonUnitario, t=0..10, scaling=CONSTRAINED)
```



```
> with(inttrans)
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(57)

[*addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier, invhilbert, invlaplace, invmellin, laplace, mellin, savetable, setup*] (57)

> EscalonTL := laplace(EscalonUnitario, t, s)

$$\text{EscalonTL} := \frac{e^{-6s}}{s} \quad (58)$$

> UnoTL := laplace(1, t, s)

$$\text{UnoTL} := \frac{1}{s} \quad (59)$$

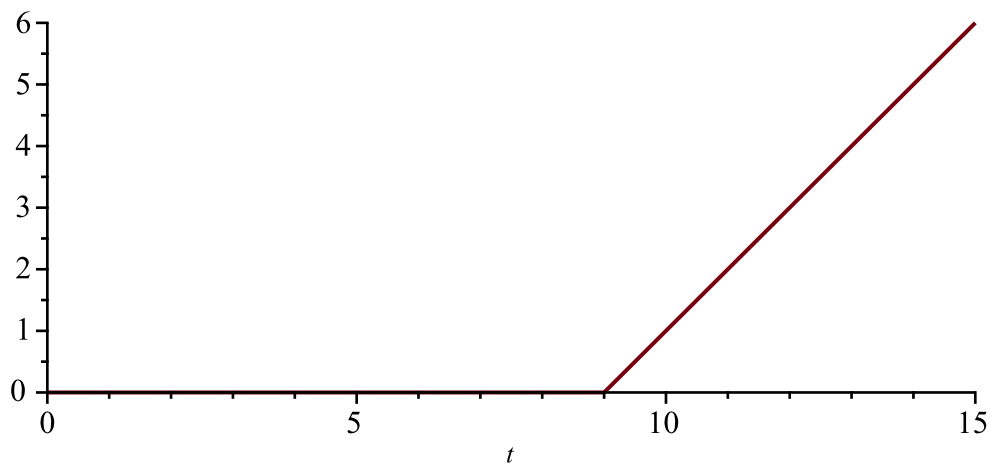
> EscalonDos := invlaplace(EscalonTL, s, t)

$$\text{EscalonDos} := \text{Heaviside}(t - 6) \quad (60)$$

> RampaUnitaria := (t - 9) · Heaviside(t - 9)

$$\text{RampaUnitaria} := (t - 9) \text{Heaviside}(t - 9) \quad (61)$$

> plot(RampaUnitaria, t = 0 .. 15, scaling = CONSTRAINED)



> RampaTL := laplace(RampaUnitaria, t, s)

$$\text{RampaTL} := \frac{e^{-9s}}{s^2} \quad (62)$$

> DosTL := laplace(t, t, s)

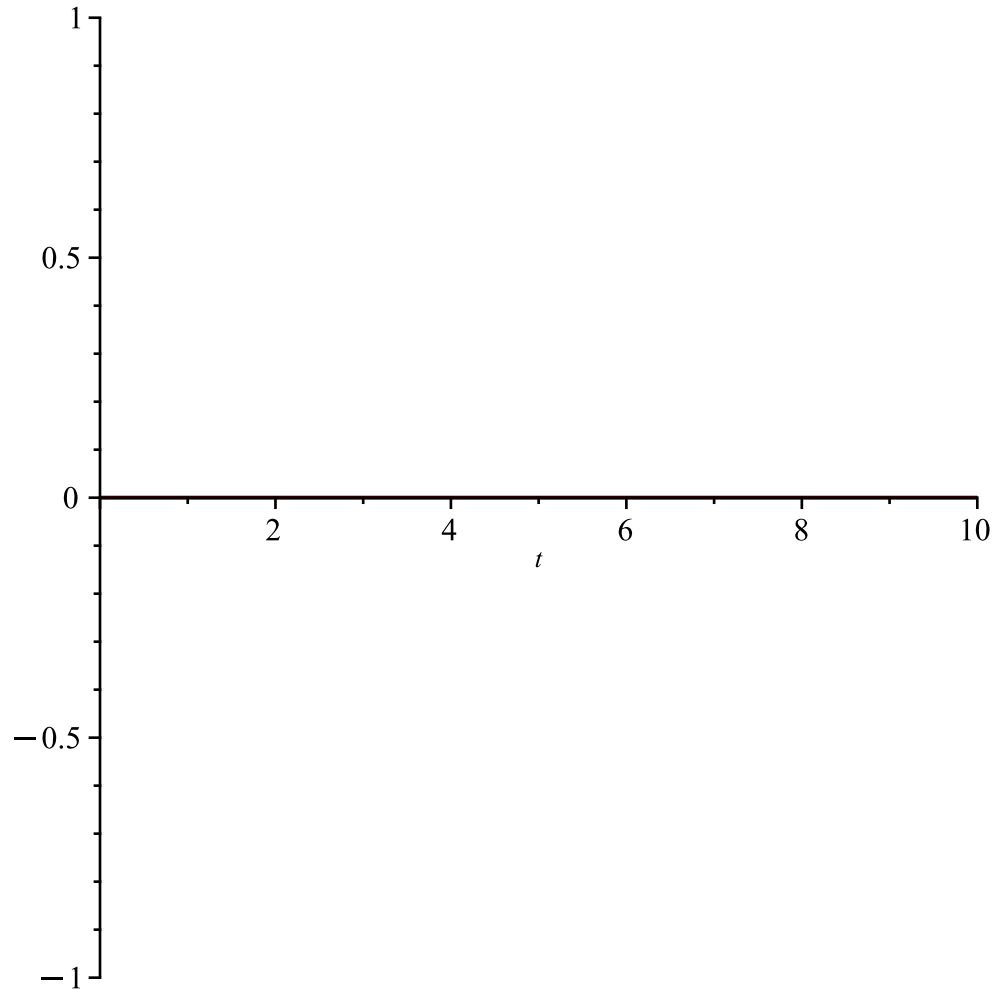
(63)

$$DosTL := \frac{1}{s^2} \quad (63)$$

```
> DeltaDirac := Dirac(t - 5)
```

$$DeltaDirac := \text{Dirac}(t - 5) \quad (64)$$

```
> plot(DeltaDirac, t = 0 .. 10)
```



```
> DeltaTL := laplace(DeltaDirac, t, s)
```

$$DeltaTL := e^{-5s} \quad (65)$$

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